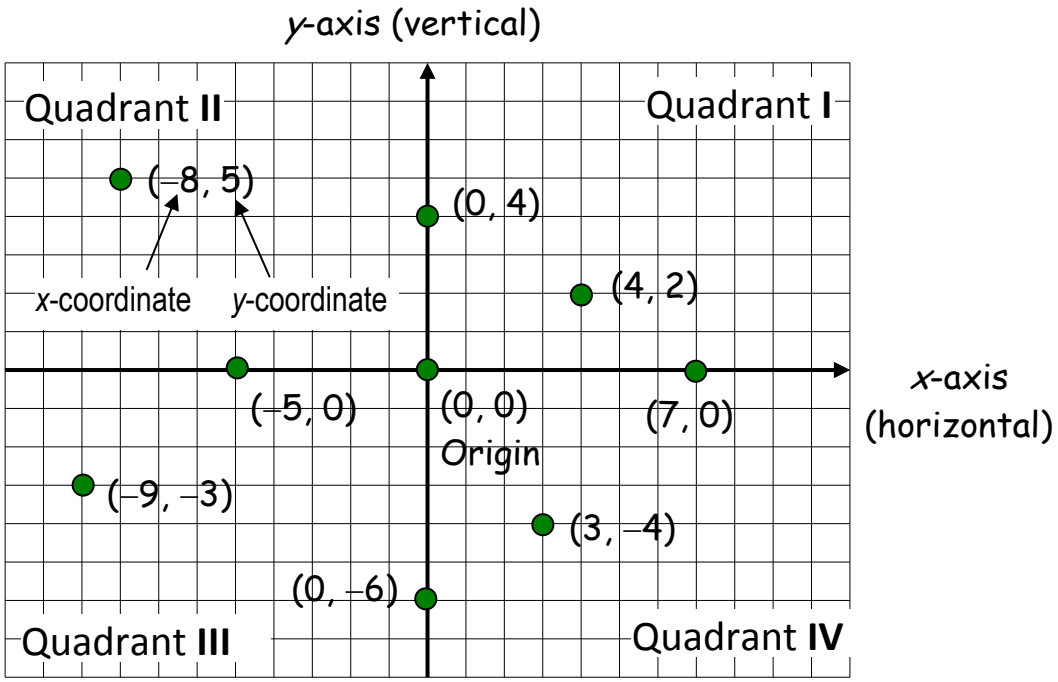
CH 1 – INTRO TO GRAPHING

❑ INTRODUCTION

A picture is worth a thousand words. This is especially true in math, where many ideas are very abstract. The French mathematician-philosopher René Descartes (who said, "I think, therefore I am") devised a way for us to visually represent the solutions of certain kinds of equations. It's called the ***Cartesian Coordinate System***, the term Cartesian being the Latin form of the name Descartes.

To create the coordinate system we take two number lines (one called the *x*-axis and one called the *y*-axis, line them up perpendicular to each other (90° angle between them), and we have a two-dimensional coordinate system.

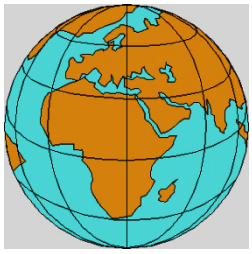
❑ THE CARTESIAN COORDINATE SYSTEM



Observations on the Cartesian Coordinate System

1. A two-dimensional coordinate system (like the figure above) represents a ***plane***. The horizontal axis is called the ***x*-axis** in math, but will be called other things in other subjects. Similarly, the vertical ***y*-axis** will be called something else in other subjects.

2. The ***ordered pair* (*x*, *y*)** represents a single ***point*** in the plane. The numbers x and y in the ordered pair are the ***coordinates*** of the point. Notice that a single point (ordered pair) consists of two coordinates. The coordinates of a point on the Earth are called its longitude and latitude.



3. The point (0, 0), where the axes intersect (cross), is called the ***origin***.

4. The first coordinate of the point (x, y) represents the distance to the right or left from the origin. The second coordinate represents the distance up or down. For example, the point $(3, -4)$ is plotted by starting at the origin, moving 3 units to the right, and then moving 4 units down.

5. The ***quadrants*** are numbered I (one) through IV (four), starting in the upper-right region and going counterclockwise.

   In Quadrant I, both coordinates (the x and y) are positive.
   In Quadrant II, x is negative and y is positive.
   In Quadrant III, both coordinates are negative.
   In Quadrant IV, x is positive and y is negative.

6. Every point on the x-axis has a y-coordinate of 0.
   Every point on the y-axis has an x-coordinate of 0.

Homework

1. Is the Cartesian coordinate system described in this chapter 1-, 2-, or 3-dimensional?

2. a. The point (7, –3) lies in Quadrant ____.
   b. The point (–8, –9) lies in Quadrant ____.
   c. The point (–1, 6) lies in Quadrant ____.
   d. The point (π, 17) lies in Quadrant ____.

3. a. The point (17, 0) lies on the ___ axis.
   b. The point (0, –20) lies on the ___ axis.
   c. The point (0, 0) is called the ____ and lies on the ___ axis.

4. a. In which quadrants are the signs of x and y the same?
   b. In which quadrants are the signs of x and y opposites?

5. a. A point lies on the x-axis. What can you say for sure about the coordinates of the point? Hint: The following are points on the x-axis: (7, 0), (–23, 0) and (π, 0).

   b. A point lies on the y-axis. What can you say for sure about the coordinates of the point?

   c. A point lies on both axes. What can you say for sure about the coordinates of the point?

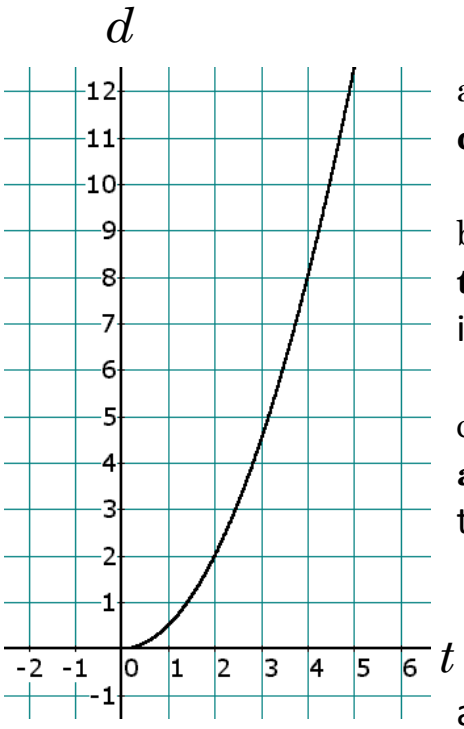
6. Does the notation (2, –7) represent two points or one point containing two coordinates?

❑ GETTING AWAY FROM X AND Y

We call the horizontal and vertical axes the x-axis and the y-axis. Why use these letters? x and y are good algebra letters — they don't stand for anything in particular, so they can represent anything. But outside this classroom, people need better variables for the things they're analyzing.

For example, when graphing degrees Fahrenheit and degrees Celsius, we might use F and C as the names of the axes. For a supply and demand curve from economics, why use x and y when we can use S and D?

EXAMPLE 1: **The graph below shows the distance in meters (d) a car has traveled based on how long it's been traveling in seconds (t).**



a. **Is the horizontal axis time or distance?** time

b. **As time increased, did the distance the car traveled increase or decrease?** increase

c. **What distance did the car travel after 4 sec?** When $t = 4$, $d = 8$, so the car traveled 8 m.

d. **About how many seconds did it take the car to travel 6 m?** Looking at the d-axis and locating the value $d = 6$, we see that the t-value is about 3.4. So it took about 3.4 sec to travel the 6 m.

Homework

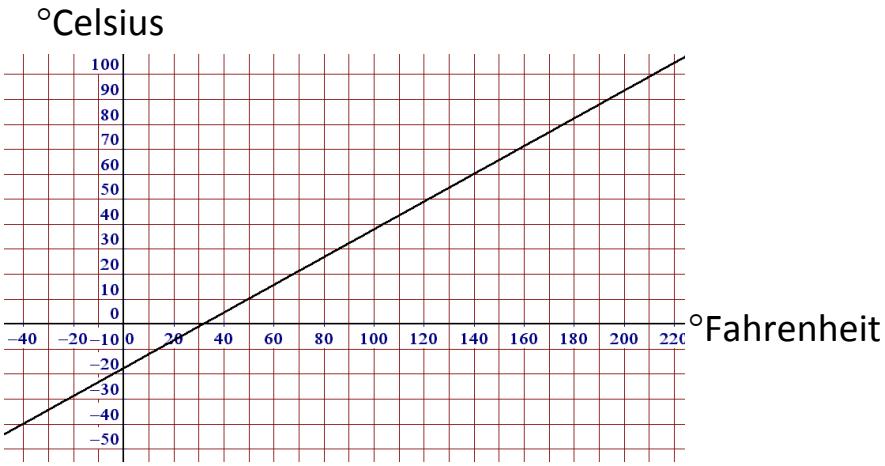
7. This problem is a continuation of Example 1.

   a. After 1 second, did the car move more than 1 m or less than 1 m?

   b. Why do you think there is no graph in the second quadrant?

   c. How far did the car travel between $t = 3$ and $t = 5$?

8. The graph below depicts the relationship between two temperature scales, Fahrenheit and Celsius.



   a. The Fahrenheit temperatures are on the (horizontal, vertical) axis.

   b. The Celsius temperatures are on the (horizontal, vertical) axis.

   c. The graph passes through every quadrant except _____.

   d. A temperature of 140°F corresponds to what temperature on the Celsius scale? Hint: Locate 140°F on the horizontal axis; then go straight up until you hit the line. Now go to the left until you hit the vertical axis. What Celsius temperature do you see?

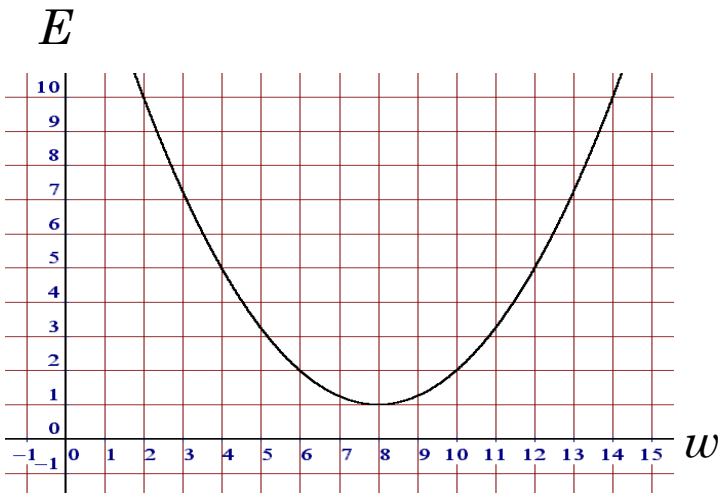
   e. What Celsius temperature corresponds to 50°F?

f. 212°F is the boiling point of water (assuming pure water and normal atmospheric pressure). Use the graph to estimate the boiling point of water on the Celsius scale.

g. 0°C is the freezing point of water. Use the graph to estimate the freezing point of water on the Fahrenheit scale.

h. This one's kind of hard: There's exactly one point on the line where the Fahrenheit and Celsius temperatures match. What temperature is that?

9. The following graph shows the relationship between E, the expenses, and w, the number of widgets sold.



a. Find the expense when 2 widgets are sold.

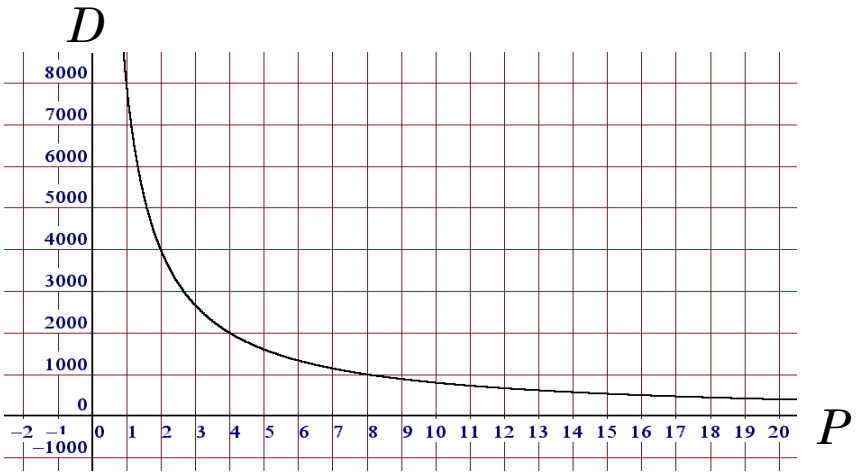
b. Find the expense when 8 widgets are sold.

c. Find the expense when 12 widgets are sold.

d. If the expense is \$2, what are the two possible values for the number of widgets sold?

e. What production level (what number of widgets) would produce the least expense?

10. The next graph shows the relationship between the demand for a deluxe widget and its price:



a. If the price of a deluxe widget is \$4, what is the demand?

b. When the price is just \$1, how many widgets are expected to be sold?

c. If the price jumps to \$16, what is the expected demand?

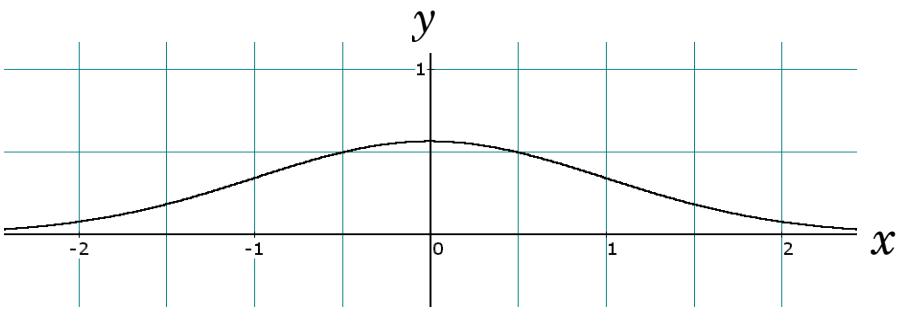
d. If the demand is 8000 widgets, what is the price of a widget?

e. As the price increases, the demand _____.

f. As the price decreases, the demand _____.

g. The entire graph is contained within which quadrant?

11. Now we come to the famous *bell-shaped curve*, one of the most important graphs in probability, statistics, business, economics, psychology, biology, and all the physical sciences.



a. When $x = 0$, y is a little bigger than ___.

b. The graph reaches its maximum y-value when $x =$ ___.

c. When $x = 0.5$, y is approximately ___.

d. When $x = -0.5$, y is approximately ___.

e. Assume that the point (3, 0.006) lies on the bell-shaped curve. What is the y-value when $x = -3$?

f. Assume that the graph goes forever to the right and forever to the left. As x moves farther and farther to the right, the y-value is always (positive, negative) and gets infinitely close to ____.

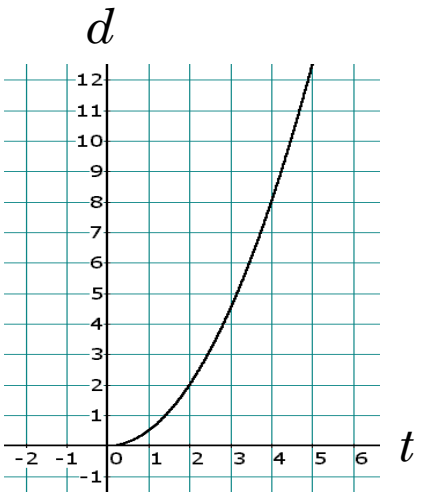
g. As x moves farther and farther to the left, the y-value is always (positive, negative) and gets infinitely close to ____.

h. The graph has symmetry with respect to which axis?

Practice Problems

12. The point $(\pi, -\pi)$ lies in which quadrant?

13. The graph below shows the distance in meters a car has traveled based on how long it's been traveling in seconds.



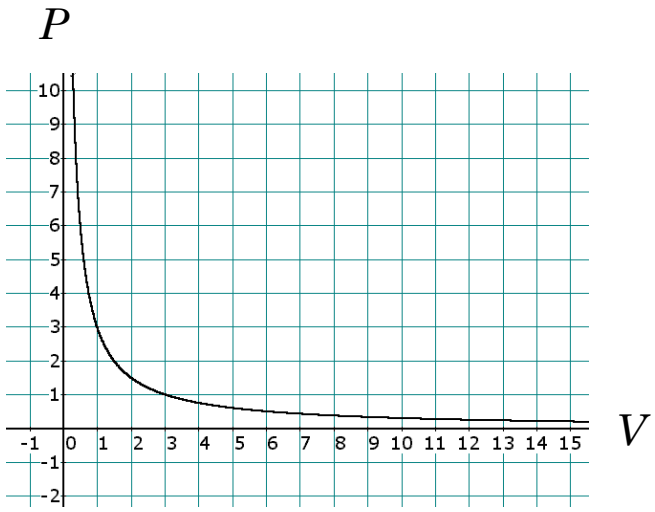
a. After 5 seconds, the car had traveled _____ meters.

b. Is the vertical axis time or distance?

c. After 3 seconds, the car had traveled _____ meters.

d. Between $t = 2$ and $t = 4$, how far did the car travel?

14. The following graph shows the relationship between the pressure and the volume of a gas:



a. When the volume is 3, the pressure is ____.
b. When the volume is 6, the pressure is ____.
c. As the volume increases, the pressure _____.
d. As the volume decreases, the pressure _____.

Solutions

1. The Cartesian Coordinate System in this course is 2-dimensional. In later courses you will encounter a 3-dimensional coordinate system, and if you major in math, physics, or computer science, you will find even higher-dimensional coordinate systems.

2. a. IV b. III c. II d. I

3. a. x b. y c. origin; it's on <u>both</u> axes.

4. a. I and III b. II and IV

5. a. that the y-coordinate must be 0.
b. that the x-coordinate must be 0.
c. both coordinates must be 0, since it's the origin.

6. one point containing two coordinates

7. a. Looking at the point where $t = 1$, we see that the d-value is between 0 and 1, so the car moved less than 1 m.

b. A point in the second quadrant would indicate a negative time value, and we usually measure time starting at zero.

c. The distance traveled at $t = 3$ is around 4.5 m, and the distance at $t = 5$ is about 12.5. Thus, the distance traveled between the two times was about 8 m.

8. a. horizontal b. vertical c. II d. 60°C e. 10°C f. 100°C
g. 32°F h. –40°

9. a. \$10 b. \$1 c. \$5 d. 6, 10 e. 8

10. a. 2000 widgets b. 8000 widgets c. 500 widgets d. \$1
e. decreases f. increases g. I

11. a. 0.5 b. 0 c. 0.5 d. 0.5 e. 0.006
f. positive; 0 g. positive; 0 h. y-axis

12. IV

13. a. about 12 b. distance c. 4.5 d. 6 m

14. a. 1 b. 0.5 c. decreases d. increases

"A well-educated mind will always have more questions than answers."

Helen Keller (1880 – 1968)